

50 Beispielaufgaben zur Potenzrechnung

1.

$$11a^3 + 5a^3 - 10a^3 = (11 + 5 - 10)a^3 = 6a^3$$

2.

$$\begin{aligned} 4\frac{1}{2}a^4 - 3\frac{2}{3}a^3 - 3\frac{1}{4}a^4 + 6\frac{3}{5}a^3 &= \frac{9}{2}a^4 - \frac{11}{3}a^3 - \frac{13}{4}a^4 + \frac{33}{5}a^3 \\ &= \frac{9}{2}a^4 - \frac{13}{4}a^4 - \frac{11}{3}a^3 + \frac{33}{5}a^3 \\ &= \left(\frac{9}{2} - \frac{13}{4}\right)a^4 + \left(\frac{33}{5} - \frac{11}{3}\right)a^3 \\ &= \frac{2 \cdot 9 - 13}{4}a^4 + \frac{3 \cdot 33 - 5 \cdot 11}{15}a^3 \\ &= \frac{5}{4}a^4 + \frac{44}{15}a^3 \\ &= 1\frac{1}{4}a^4 + 2\frac{14}{15}a^3 \end{aligned}$$

3.

$$\begin{aligned} 16a^4b - 3a^3b^2 - (7a^3b^2 - 6a^4b) &= 16a^4b - 3a^3b^2 - 7a^3b^2 + 6a^4b \\ &= 16a^4b + 6a^4b - 3a^3b^2 - 7a^3b^2 \\ &= 22a^4b - 10a^3b^2 \end{aligned}$$

4.

$$\begin{aligned} 17x^5y^7 - (3x^4z - 6x^4z) + 4x^5y^7 + 3x^4z &= 17x^5y^7 - 3x^4z + 6x^4z + 4x^5y^7 + 3x^4z \\ &= 17x^5y^7 + 4x^5y^7 - 3x^4z + 6x^4z + 3x^4z \\ &= 21x^5y^7 + 6x^4z \end{aligned}$$

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$$\begin{aligned} 17x^5y^7 - (3x^4z - 6x^4z) + 4x^5y^7 + 3x^4z &= 17x^5y^7 - (-3x^4z) + 4x^5y^7 + 3x^4z \\ &= 17x^5y^7 + 3x^4z + 4x^5y^7 + 3x^4z \\ &= 17x^5y^7 + 4x^5y^7 + 3x^4z + 3x^4z \\ &= 21x^5y^7 + 6x^4z \end{aligned}$$

5.

$$n^4 \cdot n^7 = n^{4+7} = n^{11}$$

6.

$$\begin{aligned} 2a^2b^3 \cdot 3a^4b^5 &= 2 \cdot 3a^2a^4b^3b^5 \\ &= 6a^{2+4}b^{3+5} \\ &= 6a^6b^8 \end{aligned}$$

7.

$$\begin{aligned}\frac{1}{3}a^3b^4c^2 \cdot \frac{3}{4}a^3b^5c^6 &= \frac{1}{3} \cdot \frac{3}{4}a^3a^3b^4b^5c^2c^6 \\ &= \frac{1 \cdot 3}{3 \cdot 4}a^{3+3}b^{4+5}c^{2+6} \\ &= \frac{1}{4}a^6b^9c^8\end{aligned}$$

8.

$$\begin{aligned}(x^2 - 2) \cdot 3x^3 &= x^2 \cdot 3x^3 - 2 \cdot 3x^3 \\ &= 3x^2x^3 - 6x^3 \\ &= 3x^{2+3} - 6x^3 \\ &= 3x^5 - 6x^3\end{aligned}$$

9.

$$\begin{aligned}a^{3x} \cdot a^x &= a^{3x+x} \\ &= a^{4x}\end{aligned}$$

10.

$$\begin{aligned}x^{3a} \cdot x &= x^{3a} \cdot x^1 \\ &= x^{3a+1}\end{aligned}$$

11.

$$\begin{aligned}a^{x+y} \cdot a^{x-y} &= a^{(x+y)+(x-y)} \\ &= a^{x+y+x-y} \\ &= a^{2x}\end{aligned}$$

12.

$$\begin{aligned}n^{6x+a} \cdot n^{2x-2a} &= n^{(6x+a)+(2x-2a)} \\ &= n^{6x+a+2x-2a} \\ &= n^{8x-a}\end{aligned}$$

13.

$$\begin{aligned}(n+a)^{2x+3y} \cdot (n+a)^{4x-2y} &= (n+a)^{(2x+3y)+(4x-2y)} \\ &= (n+a)^{2x+3y+4x-2y} \\ &= (n+a)^{6x+y}\end{aligned}$$

14.

$$(a+b)^a \cdot (a+b)^b = (a+b)^{a+b}$$

15.

$$\begin{aligned}
2n^{3x-2a} \cdot n^{x+a} + 3a^{2x-3y} \cdot 5a^{3x+y} &= 2n^{(3x-2a)+(x+a)} + 3 \cdot 5a^{(2x-3y)+(3x+y)} \\
&= 2n^{3x-2a+x+a} + 15a^{2x-3y+3x+y} \\
&= 2n^{4x-a} + 15a^{5x-2y}
\end{aligned}$$

16.

$$\begin{aligned}
8a^{6-5y} \cdot 3a^{2+6y} - 5a^{4+y} \cdot 6a^{2+3y} - (24a^{8+y} - 10a^{6+4y}) \\
&= 24a^{6-5y+2+6y} - 30a^{4+y+2+3y} - 24a^{8+y} + 10a^{6+4y} \\
&= 24a^{8+y} - 30a^{6+4y} - 24a^{8+y} + 10a^{6+4y} \\
&= 24a^{8+y} - 24a^{8+y} - 30a^{6+4y} + 10a^{6+4y} \\
&= (24 - 24)a^{8+y} + (-30 + 10)a^{6+4y} \\
&= 0a^{8+y} + (-20)a^{6+4y} \\
&= -20a^{6+4y}
\end{aligned}$$

17.

$$\begin{aligned}
2a^2b^3c^4 \cdot (4a^3x - 6a^2x^2y + 7x^4y) \\
&= 2a^2b^3c^4 \cdot 4a^3x - 2a^2b^3c^4 \cdot 6a^2x^2y + 2a^2b^3c^4 \cdot 7x^4y \\
&= 8a^2a^3b^3c^4x - 12a^2a^2b^3c^4x^2y + 14a^2b^3c^4x^4y \\
&= 8a^5b^3c^4x - 12a^4b^3c^4x^2y + 14a^2b^3c^4x^4y
\end{aligned}$$

18.

$$\begin{aligned}
(5a^4 + 3xy^2) \cdot (5a^4 - 3xy^2) &= 5a^4 \cdot 5a^4 - 3xy^2 \cdot 3xy^2 \\
&= 25a^8 - 9x^2y^4
\end{aligned}$$

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$$\begin{aligned}
(5a^4 + 3xy^2) \cdot (5a^4 - 3xy^2) &= (5a^4)^2 - (3xy^2)^2 \\
&= 5^2 \cdot (a^4)^2 - 3^2 \cdot x^2 \cdot (y^2)^2 \\
&= 25a^{4 \cdot 2} - 9x^2y^{2 \cdot 2} \\
&= 25a^8 - 9x^2y^4
\end{aligned}$$

19.

$$\begin{aligned}
(3abc)^4 &= 3^4a^4b^4c^4 \\
&= 81a^4b^4c^4
\end{aligned}$$

20.

$$\begin{aligned}
\frac{3}{4} \cdot (4ab)^4 &= \frac{3}{4} \cdot 4^4a^4b^4 \\
&= 3 \cdot 4^{-1}4^4a^4b^4 \\
&= 3 \cdot 4^{-1+4}a^4b^4 \\
&= 3 \cdot 4^3a^4b^4 \\
&= 192a^4b^4
\end{aligned}$$

21.

$$\begin{aligned}25x^2 \cdot (5xy)^2 &= 25x^2 \cdot 5^2x^2y^2 \\ &= 25 \cdot 25x^4y^2 \\ &= 625x^4y^2\end{aligned}$$

22.

$$\begin{aligned}(2a^3) : (2a^5) &= \frac{2a^3}{2a^5} \\ &= \frac{\cancel{2}a^3a^{-5}}{\cancel{2}} \\ &= a^{3+(-5)} \\ &= a^{-2} \\ &= \frac{1}{a^2}\end{aligned}$$

23.

$$\begin{aligned}(n+a) : (n+a)^3 &= \frac{(n+a)^1}{(n+a)^3} \\ &= (n+a)^{1-3} \\ &= (n+a)^{-2} \\ &= \frac{1}{(n+a)^2}\end{aligned}$$

24.

$$\begin{aligned}(4n^7x^8) : (8n^3x^4) &= \frac{4n^7x^8}{8n^3x^4} \\ &= \frac{\cancel{4}n^7x^8n^{-3}x^{-4}}{\cancel{8}} \\ &= \frac{n^{7+(-3)}x^{8-4}}{2} \\ &= \frac{n^4x^4}{2}\end{aligned}$$

25.

$$\begin{aligned}(15x^3n^2b) : (5x^4n^3b^2) &= \frac{15x^3n^2b}{5x^4n^3b^2} \\ &= \frac{\cancel{15}x^3n^2b^{\cancel{2}}}{\cancel{5}x^4n^3b^2x^{-3}n^{-2}b^{-1}} \\ &= \frac{3}{x^{4-3}n^{3-2}b^{2-1}} \\ &= \frac{3}{xnb}\end{aligned}$$

26.

$$\begin{aligned} \frac{5a^2b}{n^4x^5} - \frac{9c^3n}{n^3x^4} &= \frac{5a^2b}{n^4x^5} - \frac{9c^3n \cdot nx}{n^3x^4 \cdot nx} \\ &= \frac{5a^2b}{n^4x^5} - \frac{9c^3n^2x}{n^4x^5} \\ &= \frac{5a^2b - 9c^3n^2x}{n^4x^5} \end{aligned}$$

27.

$$\begin{aligned} \frac{14a^4b^5x}{24ab^7x} - \frac{9a^3b}{13a^2b^3} + \frac{ab^3}{2b^{10}} &= \frac{7a^{4-1}}{12b^{7-5}} - \frac{9a^{3-2}}{13b^{3-1}} + \frac{a}{2b^{10-3}} \\ &= \frac{7a^3}{12b^2} - \frac{9a}{13b^2} + \frac{a}{2b^7} \\ &= \frac{7a^3 \cdot 13b^5}{12b^2 \cdot 13b^5} - \frac{9a \cdot 12b^5}{13b^2 \cdot 12b^5} + \frac{a \cdot (6 \cdot 13)}{2b^7 \cdot (6 \cdot 13)} \\ &= \frac{91a^3b^5}{156b^7} - \frac{108ab^5}{156b^7} + \frac{78a}{156b^7} \\ &= \frac{91a^3b^5 - 108b^5 + 78a^2}{156ab^7} \end{aligned}$$

28.

$$\begin{aligned} \frac{2b^2}{3a^2} \cdot \frac{15a}{12b^4} &= \frac{2b^2 \cdot 15a}{3a^2 \cdot 12b^4} \\ &= \frac{\cancel{2} \cdot \cancel{15} ab^2}{\cancel{3} \cdot \cancel{12} a^2b^4} \\ &= \frac{5}{6a^{2-1}b^{4-2}} \\ &= \frac{5}{6ab^2} \end{aligned}$$

29.

$$\begin{aligned} \frac{3ab}{14xy^5} \cdot \frac{28xy^6}{12ab} &= \frac{3ab \cdot 28xy^6}{14xy^5 \cdot 12ab} \\ &= \frac{\cancel{3} \cdot \cancel{28} abxy^6}{\cancel{14} \cdot \cancel{12} abxy^5} \\ &= \frac{y^{6-5}}{2} \\ &= \frac{y}{2} \end{aligned}$$

30.

$$\begin{aligned} \left(\frac{2a}{b}\right)^3 &= \frac{(2a)^3}{b^3} \\ &= \frac{2^3a^3}{b^3} \\ &= \frac{8a^3}{b^3} \end{aligned}$$

31.

$$\begin{aligned} \left(\frac{3c}{2d}\right)^5 &= \frac{(3c)^5}{(2d)^5} \\ &= \frac{3^5 c^5}{2^5 d^5} \\ &= \frac{243c^5}{32d^5} \end{aligned}$$

32.

$$\begin{aligned} \left(\frac{2n}{3m}\right)^6 \cdot \left(\frac{3m}{n}\right)^4 &= \frac{(2n)^6}{(3m)^6} \cdot \frac{(3m)^4}{n^4} \\ &= \frac{2^6 n^6 \cdot 3^4 m^4}{3^6 m^6 \cdot n^4} \\ &= \frac{2^6 n^{6-4}}{3^{6-4} m^{6-4}} \\ &= \frac{64n^2}{9m^2} \end{aligned}$$

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$$\begin{aligned} &= \frac{8^2 n^2}{3^2 m^2} \\ &= \frac{(8n)^2}{(3m)^2} \\ &= \left(\frac{8n}{3m}\right)^2 \end{aligned}$$

33.

$$\begin{aligned} \frac{24x^3}{50a} \cdot \left(\frac{10a}{8x}\right)^5 &= \frac{24x^3}{50a} \cdot \frac{(10a)^5}{(8x)^5} \\ &= \frac{3 \cdot 8x^3}{5 \cdot 10a} \cdot \frac{10^5 a^5}{8^5 x^5} \\ &= \frac{3 \cdot 8x^3 \cdot 10^5 a^5}{5 \cdot 10a \cdot 8^5 x^5} \\ &= \frac{3 \cdot 10^{5-1} a^{5-1}}{5 \cdot 8^{5-1} x^{5-3}} \\ &= \frac{3 \cdot 10^4 a^4}{5 \cdot 8^4 x^2} \\ &= \frac{3 \cdot (2 \cdot 5)^4 a^4}{5 \cdot (2 \cdot 4)^4 x^2} \\ &= \frac{3 \cdot 2^4 \cdot 5^4 a^4}{5 \cdot 2^4 \cdot 4^4 x^2} \\ &= \frac{3 \cdot 5^{4-1} a^4}{4^4 x^2} \\ &= \frac{375a^4}{256x^2} \end{aligned}$$

34.

$$\begin{aligned}
\frac{n^4}{x^3} \div \frac{n^5 b^2}{cx^5} &= \frac{n^4}{x^3} \cdot \frac{cx^5}{n^5 b^2} \\
&= \frac{n^4 \cdot cx^5}{x^3 \cdot n^5 b^2} \\
&= \frac{cx^{5-3}}{n^{5-4} b^2} \\
&= \frac{cx^2}{nb^2}
\end{aligned}$$

35.

$$\begin{aligned}
&\left(\frac{13a^7}{10x^5} \div \frac{26a^4}{30x^8}\right) - \left(\frac{24a^3c}{13cx^4} \div \frac{8}{26x^7}\right) \\
&= \left(\frac{13a^7}{10x^5} \cdot \frac{30x^8}{26a^4}\right) - \left(\frac{24a^3c}{13cx^4} \cdot \frac{26x^7}{8}\right) \\
&= \frac{13a^7 \cdot 30x^8}{10x^5 \cdot 26a^4} - \frac{324a^3c \cdot 26x^7}{13cx^4 \cdot 18} \\
&= \frac{3a^7x^8}{2a^4x^5} - \frac{6a^3cx^7}{cx^4} \\
&= \frac{3a^{7-4}x^{8-5}}{2} - \frac{6a^3x^{7-4}}{1} \\
&= \frac{3}{2}a^3x^3 - 6a^3x^3 \\
&= -\frac{9}{2}a^3x^3
\end{aligned}$$

36.

$$\begin{aligned}
\frac{3a^3b^2}{4n^{-2}d^6} \cdot \frac{3a^{-4}b^3}{d^3n^2} &= \frac{3a^3b^2 \cdot 3a^{-4}b^3}{4n^{-2}d^6 \cdot d^3n^2} \\
&= \frac{9a^{3-4}b^{2+3}}{4n^{-2+2}d^{6+3}} \\
&= \frac{9a^{-1}b^5}{4d^9} \\
&= \frac{9b^5}{4ad^9}
\end{aligned}$$

37.

$$\begin{aligned}
\frac{n^{4a-2b}}{n^{2a+2b}} &= n^{(4a-2b)-(2a+2b)} \\
&= n^{4a-2b-2a-2b} \\
&= n^{2a-4b}
\end{aligned}$$

38.

$$\begin{aligned}
\frac{8a^{2x+6}}{24a^{2x-6}} &= \frac{1}{3}a^{(2x+6)-(2x-6)} \\
&= \frac{1}{3}a^{2x+6-2x+6} \\
&= \frac{a^{12}}{3}
\end{aligned}$$

39.

$$\begin{aligned}
 \frac{b^{7x+5y}}{b^{4x-5y}} - \frac{b^{6y+8x}}{b^{5x-4y}} &= b^{(7x+5y)-(4x-5y)} - b^{(6y+8x)-(5x-4y)} \\
 &= b^{7x+5y-4x+5y} - b^{6y+8x-5x+4y} \\
 &= b^{3x+10y} - b^{10y+3x} \\
 &= 0
 \end{aligned}$$

40.

$$\begin{aligned}
 &\frac{6b^{x+y} - 10b^{y+2z-x} - 12b^{x-z} + 20b^{z-x}}{3b^{x-z} - 5b^{z-x}} \\
 &= \frac{6b^{(y+z)+(x-z)} - 10b^{(y+z)+(z-x)} - 4(3b^{x-z} - 5b^{z-x})}{3b^{x-z} - 5b^{z-x}} \\
 &= \frac{6b^{y+z}b^{x-z} - 10b^{y+z}b^{z-x} - 4(3b^{x-z} - 5b^{z-x})}{3b^{x-z} - 5b^{z-x}} \\
 &= \frac{2b^{y+z}(3b^{x-z} - 5b^{z-x}) - 4(3b^{x-z} - 5b^{z-x})}{3b^{x-z} - 5b^{z-x}} \\
 &= \frac{(2b^{y+z} - 4) \cdot (3b^{x-z} - 5b^{z-x})}{3b^{x-z} - 5b^{z-x}} \\
 &= 2b^{y+z} - 4
 \end{aligned}$$

41.

$$\begin{aligned}
 (3x^2y^3)^2 &= 3^2(x^2)^2(y^3)^2 \\
 &= 9x^{2 \cdot 2}y^{3 \cdot 2} \\
 &= 9x^4y^6
 \end{aligned}$$

42.

$$\begin{aligned}
 (b^x)^{-n} &= b^{x \cdot (-n)} \\
 &= b^{-nx}
 \end{aligned}$$

43.

$$\begin{aligned}
 \left(\frac{4}{7}a^2\right)^{-3} &= \left(\frac{4}{7}\right)^{-3} (a^2)^{-3} \\
 &= \left(\frac{7}{4}\right)^3 a^{2 \cdot (-3)} \\
 &= \frac{7^3}{4^3} a^{-6} \\
 &= \frac{343}{64} \cdot \frac{1}{a^6} \\
 &= \frac{343}{64a^6}
 \end{aligned}$$

44.

$$\begin{aligned}
\left(-\frac{2a^2bx}{5c}\right)^{-4} &= \left(-\frac{5c}{2a^2bx}\right)^4 \\
&= (-1)^4 \cdot \left(\frac{5c}{2a^2bx}\right)^4 \\
&= \frac{(5c)^4}{(2a^2bx)^4} \\
&= \frac{5^4c^4}{2^4(a^2)^4b^4x^4} \\
&= \frac{625c^4}{16a^{2 \cdot 4}b^4x^4} \\
&= \frac{625c^4}{16a^8b^4x^4}
\end{aligned}$$

45.

$$\begin{aligned}
\left(\frac{a^{-3}b^4}{c^2x^{-2}b^0}\right)^{-3} &= \frac{(a^{-3}b^4)^{-3}}{(c^2x^{-2}b^0)^{-3}} \\
&= \frac{(a^{-3})^{-3}(b^4)^{-3}}{(c^2)^{-3}(x^{-2})^{-3}} \\
&= \frac{a^{(-3) \cdot (-3)}b^{4 \cdot (-3)}}{c^{2 \cdot (-3)}x^{(-2) \cdot (-3)}} \\
&= \frac{a^9b^{-12}}{c^{-6}x^6} \\
&= \frac{a^9c^6}{b^{12}x^6}
\end{aligned}$$

46.

$$\begin{aligned}
\left((x^2)^3\right)^5 &= (x^2)^{3 \cdot 5} \\
&= (x^2)^{15} \\
&= x^{2 \cdot 15} \\
&= x^{30}
\end{aligned}$$

47.

$$\begin{aligned}
(a^{2x+5b})^{2x-5b} &= a^{(2x+5b) \cdot (2x-5b)} \\
&= a^{(2x)^2 - (5b)^2} \\
&= a^{4x^2 - 25b^2}
\end{aligned}$$

48.

$$\begin{aligned}
 \left(\frac{3a^2b^3}{5x^2y^3}\right)^2 \div \left(\frac{3ab^2}{5x^2y^2}\right)^3 &= \frac{(3a^2b^3)^2}{(5x^2y^3)^2} \div \frac{(3ab^2)^3}{(5x^2y^2)^3} \\
 &= \frac{3^2(a^2)^2(b^3)^2}{5^2(x^2)^2(y^3)^2} \div \frac{3^3a^3(b^2)^3}{5^3(x^2)^3(y^2)^3} \\
 &= \frac{3^2a^4b^6}{5^2x^4y^6} \div \frac{3^3a^3b^6}{5^3x^6y^6} \\
 &= \frac{3^2a^4b^6}{5^2x^4y^6} \cdot \frac{5^3x^6y^6}{3^3a^3b^6} \\
 &= \frac{3^2 \cdot 5^3 a^4 b^6 x^6 y^6}{5^2 \cdot 3^3 a^3 b^6 x^4 y^6} \\
 &= \frac{5^{3-2} a^{4-3} b^{6-6} x^{6-4} y^{6-6}}{3^{3-2}} \\
 &= \frac{5}{3} ax^2
 \end{aligned}$$

49.

$$\begin{aligned}
 \left(\frac{2a^4}{3b^5}\right)^3 \cdot \left(\frac{b^2}{a^2}\right)^2 \div \left(\frac{2a}{3b}\right)^3 &= \frac{(2a^4)^3}{(3b^5)^3} \cdot \frac{(b^2)^2}{(a^2)^2} \div \frac{(2a)^3}{(3b)^3} \\
 &= \frac{2^3(a^4)^3}{3^3(b^5)^3} \cdot \frac{b^4}{a^4} \cdot \frac{3^3b^3}{2^3a^3} \\
 &= \frac{2^3 \cdot 3^3 a^{12} \cdot b^4 \cdot b^3}{3^3 \cdot 2^3 b^{15} \cdot a^4 \cdot a^3} \\
 &= \frac{a^{12} \cdot b^7}{b^{15} a^7} \\
 &= \frac{a^5}{b^8}
 \end{aligned}$$

50.

$$\begin{aligned}
 \frac{((5a)^x)^{3b}}{(5a)^{2bx} \cdot (4c)^{bx}} &= \frac{(5a)^{x \cdot 3b}}{(5a)^{2bx} (4c)^{bx}} \\
 &= \frac{(5a)^{3bx-2bx}}{(4c)^{bx}} \\
 &= \frac{(5a)^{bx}}{(4c)^{bx}} \\
 &= \left(\frac{5a}{4c}\right)^{bx}
 \end{aligned}$$